the tail. According to these admeasurements the Pelorosaurus would be 81 feet long, and its body 20 feet in circumference. But if we assume the length and number of the vertebræ as the scale, we should have a reptile of relatively abbreviated proportions; even in this case, however, the original creature would far surpass in magnitude the most colossal of reptilian forms.

In conclusion, Dr. Mantell comments on the probable physical conditions of the countries inhabited by the terrestrial reptiles of the secondary ages of geology. These highly-organized colossal land saurians appear to have occupied the same position in those ancient faunas as the large mammalia in those of modern times. The trees and plants whose remains are associated with the fossil bones, manifest, by their close affinity to living species, that the islands or continents on which they grew possessed as pure an atmosphere, as high a temperature, and as unclouded skies as those of our tropical climes. There are therefore no legitimate grounds for the hypothesis in which some physiologists have indulged, that during the "*Age of Reptiles*" the earth was in the state of a half-finished planet, and its atmosphere too heavy, from an excess of carbon, for the respiration of warm-blooded animals. Such an opinion can only have originated from a partial view of all the phenomena which these problems embrace, for there is as great a discrepancy between the existing faunas of different regions, as in the extinct groups of animals and plants which geological researches have revealed.

The memoir was illustrated by numerous drawings, and the gigantic humerus of the Pelorosaurus and other bones were placed before the Society.

February 21, 1850.

GEORGE RENNIE, Esq., Treasurer, in the Chair.

Robert Alfred Cloyne Austen, Esq. was admitted into the Society.

The following papers were read:—

1. "On the Extension of the Principle of Fermat's Theorem of the Polygonal Numbers to the higher orders of series whose ultimate differences are constant. With a new Theorem proposed, applicable to all the Orders." By Sir Frederick Pollock, Lord Chief Baron, F.R.S.

The object of this paper professes to be to ascertain whether the principle of Fermat's theorem of the polygonal numbers may not be extended to all orders of series whose ultimate differences are constant. The polygonal numbers are all of the *quadratic* form, and they have (according to Fermat's theorem) this property, that every number is the sum of not exceeding, 3 terms of the triangular numbers, 4 of the square numbers, 5 of the pentagonal numbers, &c.

It is stated in this paper that the series of the odd squares 1, 9, 25, 49, &c. has a similar property, and that every number is the sum of

not exceeding 10 odd squares. It is also stated, that a series consisting of the 1st and every succeeding 3rd term of the triangular series, viz. 1, 10, 28, 35, &c., has a similar property; and that every number is the sum of not exceeding 11 terms of this last series, and that this may be easily proved [it was proved in a former paper by the same author]. The term "Notation-limit" is applied to the number which denotes the largest number of terms of a series necessary to express any number; and the writer states that 5, 7, 9, 13, 21 are respectively the notation-limits of the tetrahedral numbers, the octohedral, the cubical, the eicosahedral and the dodecahedral numbers; that 19 is the notation-limit of the series of the 4th powers; that 11 is the notation-limit of the series of the triangular numbers squared, viz. 1, 9, 36, 100, &c., and 31 the notation-limit of the series 1, 28, 153, &c. (the sum of the odd cubes), whose general expression is $2n^4-n^2$.

The paper next contains an extension of the theorem $8n+3=3$ odd squares, which was proved by Legendre in his *Théorie des Nombres*; every odd square equals 8 times a triangular No. + 1; the theorem therefore is—8 times any term in the figurate series (1, 2, 3, 4, &c...) $+3=3$ terms of a series consisting of the next series, viz. (1, 3, 6, 10 .. &c.), multiplied by 8 with 1 prefixed, and also added to each term. But it is stated that this theorem may be much extended; for this is not only true of any two consecutive series, but generally if F_x represent any figurate number of the x^{th} order, and F_y any figurate number of the y^{th} order, whether y be greater or less than x,

$$8F_x+3=3, \text{ or } (3+8), \text{ or } (3+2\,.\,8), \text{ or} \ldots (3+n8), \text{\&c.},$$

terms of a series whose general expression is $8F_y+1$; and still further (provided p be greater than 2)—

$$pF_x+3=3, \text{ or } (3+p), \text{ or } (3+2p), \text{ or } (3+np),$$

terms of a series whose general expression is pF_y+1, and *vice versâ*.

The author concludes from these considerations, that probably there are many theorems which are common to all the orders. The following theorem is then proposed as having that character.

If the terms of a series be

$$\begin{array}{l} 1, \text{ or } (1+n)^0, (1+n)^1, (1+n)^2 \ldots \text{\&c. } (1+n)^p, \\ \qquad \text{the 1st } (p+1) \text{ terms of } (1+n)^{p+1} \\ \qquad \text{the 1st } (p+1) \text{ terms of } (1+n)^{p+2} \\ \qquad \text{the 1st } (p+1) \text{ terms of } (1+n)^{p+3} \\ \qquad\qquad + \text{ \&c. \&c.} \end{array}$$

(if p and n be both not less than 1), any number will be the sum of not exceeding $(pn+1)$ terms of the series; in other words, $pn+1$ is the notation-limit of this series.

It is manifest that this series is of such a form, that by varying n and p, it is capable of expressing every possible arithmetical series, also every possible geometrical series (each having 1 for the first term); it will also express all the intermediate series of the successive orders (to an indefinite extent), which exist between and con-

nect together by a regular gradation (as is well known) any such arithmetical series with a geometric series, whose common ratio is the 2nd term of both series. The theorem may be stated without the series thus :—

If any geometric series (having 1 for its first term) and $(1+n)$ for its common ratio, be stayed at the $(p+1)$th term and discontinued as a *geometric* series, but be continued from that term as an *arithmetic* series of the *pth* order, by forming it with the *pth* difference as the constant difference, and the other differences (which will be x, x^2, x^3, &c. . . . . x^p). The resulting series will be the series stated in the theorem above, and any number may be formed by not exceeding $(pn+1)$ terms, that is $(pn+1)$ will be the notation-limit of the series; if p becomes indefinitely great, the limit of the series is a geometrical series, and it would become capable of expressing any number according to a system of notation whose base or local value would be $(1+n)$.

The proof of the theorem seems to depend upon this, that the notation-limit assigned by the theorem is actually the notation-limit of all the geometric terms and one more, at least, while the geometric terms alone fix the law of the series and ascertain its *elements* (that is, the first term and the successive differences); and as the combinations necessary to enable the series to fulfill its law, and carry on the notation that belongs to it, are regulated by the series next below it, viz. by the first rank of differences, while the supply of new combinations (as the series advances and the number of terms that may be used increases) is indicated by even a higher series than itself, the new combinations are always greater, and at length indefinitely greater, than the number required. If therefore within the range of those terms that ascertain and fix the law of the series the law of its notation-limit can be obeyed, it must always (*à fortiori*) be obeyed as the series proceeds to a greater number of terms and to a variety of combinations increasing in a higher ratio; and the series will furnish the numbers requisite to carry on the notation by the new and more numerous combinations which must of necessity be of the same numerical kind with those which have preceded them. It is shown at length, that the new combinations, as the series advances, do actually increase in an increasing proportion compared with the numbers required.

2. "Experiments on the section of the Glossopharyngeal and Hypoglossal Nerves of the Frog, and Observations of the alterations produced thereby in the Structure of their primitive fibres." By Augustus Waller, M.D. Communicated by Professor Owen, F.R.S. &c.

After describing the natural structure of the tubular fibres of the nerves, the author states the results which he observed to follow the section of the nerves of the frog's tongue. To this organ two principal pairs of nerves are distributed; one of these, issuing from the cranium along with the pneumogastric and distributed to the fungiform papillæ, is regarded as the glossopharyngeal; the other,